Fuzzy Systems

Fuzzy Systems

PART A

- **Introduction**
- **Applications**
- **Fuzzy sets and fuzzy logic**
- **Probability and fuzzy logic**
- **Fuzzy reasoning**
- **Design of a fuzzy controller**

PART B

- **Building fuzzy systems**
- **Advantages and limitations of fuzzy systems**
- **Case Studies**

Why fuzzy systems

- Vagueness or imprecision is inherent in many real life objects or properties

  What is the definition of “warm” or "tall"?

  – There are many such imprecise concepts

- Application of hard boundaries for categorisation gives unsatisfactory results

- Ability to handle imprecision is an attribute of intelligence

- Fuzzy logic provides a methodology for reasoning using imprecise rules and assertions

- Intelligent control and decision support systems based on fuzzy logic have proved their superiority over conventional hard logic based systems

Fuzzy system applications
Fuzzy control systems
– controlling machinery

- Most renowned fuzzy control system in use - Sendai subway (since 1987)
- Japanese appliances - vacuum cleaners, washing machines, camcorders
- Fuzzy auto transmission & ABS in cars
- Fuzzy lift control system
- Fuzzy TV!

Fuzzy intelligent systems in business - making decisions

- Fuzzy expert systems are proving to be a powerful tool in business knowledge decision support

- Successfully applied in
  - Transportation
  - Managed health care
  - Financial services such as insurance risk assessment and company stability analysis
  - Product marketing and sales analysis
  - Extraction of information from databases (data mining)
  - Resource and project management

FS applications

Year	Applications
1986	8
1987	15
1988	50
1989	100
1990	150
1991	300
1992	800
1993	1500

Table: Approximate estimated numbers of commercial and industrial applications of fuzzy systems (Munakata 1994)

- Fuzzy systems are suitable for complex problems or applications that involve intuitive thinking

Fuzzy sets – the basis of fuzzy logic

- In classical logic, the boundary of a set is sharp:

  eg, all people earning $75,000 or higher are members of set *high-income earner*

  Anyone earning less than $75,000 is not

- Because of the sharpness of the set boundary, classical logic sets are known as *crisp sets*

- As the domain value (in this case, income) increases, the degree of membership in the set high-income earner remains zero, but jumps to 1 (true) as income reaches $75000

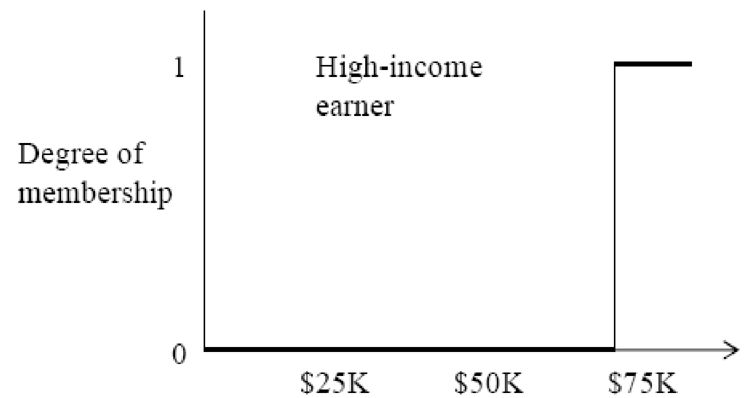


Fig. 1 membership graph for a crisp set high-income earner

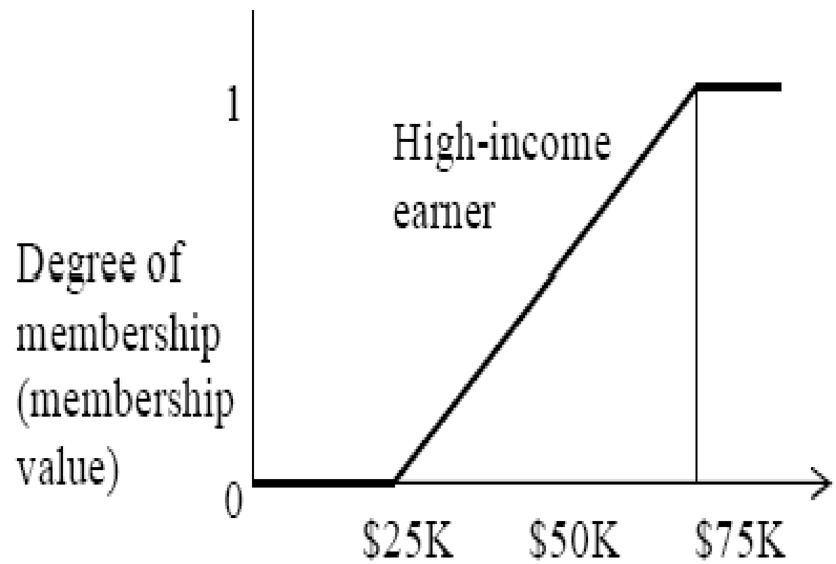
Fuzzy sets

- For a *fuzzy set*, membership values lie within the range zero (no membership) to 1 (complete membership)

- eg. the membership graph of the fuzzy set *high-income earner* may have the shape shown below

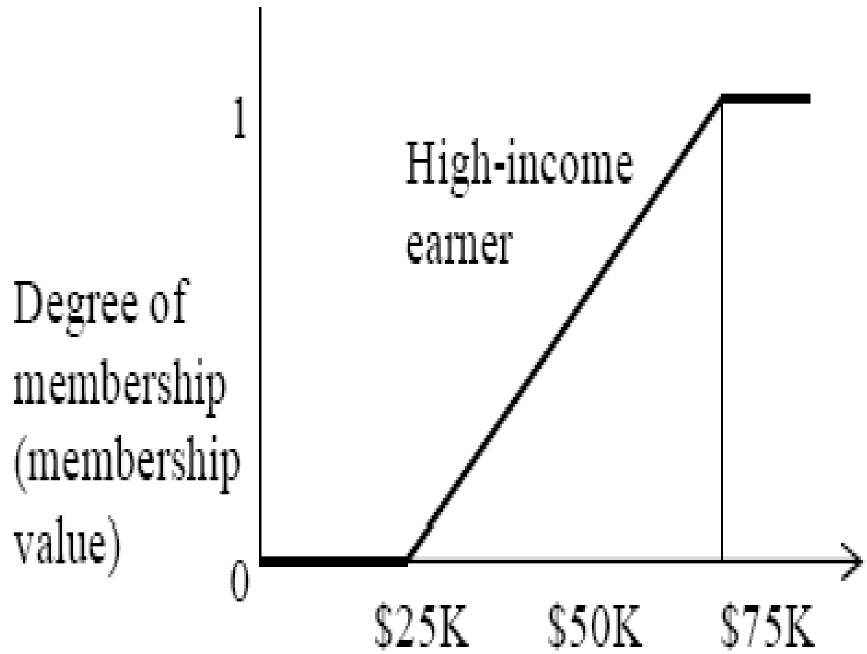
- The horizontal axis of the graphs that represent these fuzzy sets is called the *universe of discourse* over a variable of interest x

- The vertical axis is a degree of membership in the set m(x) and is always in the range [0,1]



Fuzzification

- According to this *membership function*, someone earning $30,000 will have a membership value of 0.1
- Someone earning $74,900 will have a membership value of 0.998
- This is called *fuzzification*
- All incomes at or below $25,000 have membership value 0
- All those at or above $75,000 have membership value 1



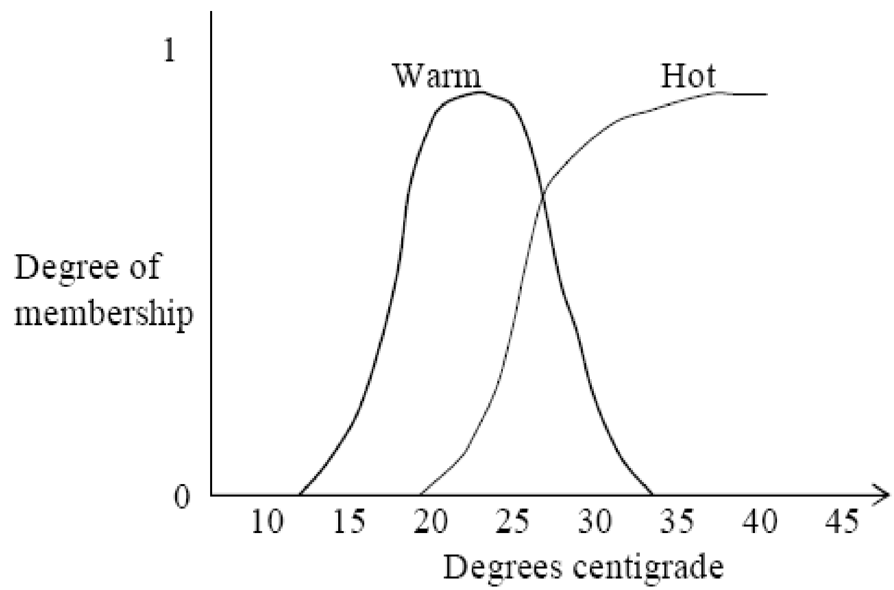
Fuzzy set examples

- Depending on the application, fuzzy set membership functions can have different shapes including S-shape, triangle, trapezoid

  eg, membership functions of fuzzy sets *warm* and *hot* are bell-shaped

- Continuous valued degrees of membership in fuzzy sets enable handling of imprecise concepts such as *high*, *weak*, *warm*, which are commonly encountered in real life problems

- In practice, these curves are often replaced by simpler triangular and trapezoidal functions, which are much faster to compute



Fuzzy logic is not just probability

- A lot of discussion about the nature o n
  the 1970s

- Many regard it as just a form of proba
  of its basis and its reliability – the nar
  has not helped

- Both fuzzy logic and probability deal v

- Both use a continuous 0 to 1 scale for

- But despite their apparent similarity,
  between the two paradigms...

Fuzzy logic and probability - the difference

- Probability deals with *likelihood* – the chance of something happening or something having a certain property
- Fuzzy logic deals not with likelihood of something having a certain property, but the *degree to which it has that property*
- The "high card" drawing example:
  P(high_card) = 16/52   (picture cards) versus

  $m_{9Hearts}$(high_cards) = 0 (picture cards) or 9/13 (linear scale)
- Fuzzy set theory and fuzzy logic provide a mathematical tool for handling this second kind of uncertainty
- Despite the associated debate, its usefulness as a powerful tool for solving problems is well established.

Fuzzy reasoning

- The fuzzy model of a problem consists of a series of unconditional and conditional fuzzy propositions

- A unconditional fuzzy proposition has the form

  x is *Y*

  where *x* is a *linguistic variable* , *Y* is the name of a fuzzy set.

  x is called a linguistic variable because its value in the proposition is expressed by a human expert using a word (linguistic expression) rather than a number

- For example, *salary* is *high*

  The truth value of this proposition is given by the degree of membership of *salary* in the fuzzy set *high*

- This membership value is computed from the actual case-specific numeric value with which *salary* is instantiated, and the fuzzy membership function *high*

Fuzzy reasoning (cont'd)

- A conditional fuzzy proposition, or rule, has the form

  IF *w* is *Z* THEN *x* is *Y*

  This should be interpreted as:

  x is a member of Y to the degree that w is a member of Z

- The consequent (RHS) of the rule is applied or executed only to the extent that the antecedent (LHS) is true

- In the example fuzzy rule

  IF *years_in_job* is *high* THEN *salary* is *high,*

- The membership value of *salary* in the fuzzy set *high* is determined by the membership value of *years_in_job* in set *high*

- The fuzzy region for the set *high* for salary will be truncated to a level determined by the truth value of the proposition *"salary* is *high"*

Inferencing through fuzzy reasoning

- A number of fuzzy propositions is evaluated for their degrees of truth
- All propositions having some truth contribute to the final output state of the solution variable
- Unlike conventional expert systems, fuzzy reasoning is based on the *parallel processing* principle
- All rules are fired even if not all of them contribute to the final outcome and some may contribute only partially

Fuzzy reasoning example

- A fuzzy rule based system for determining salary

- Rule base may consist of the rules:

  IF *years_in_job* is *high* THEN *salary* is *high*

  IF *years_in_job* is *medium* THEN *salary* is *medium*

  IF *years_in_job* is *low* THEN *salary* is *low*

  IF *products_sold* is *high* THEN *salary* is *high*

  IF *products_sold* is *medium* THEN *salary* is *medium*

  IF *products_sold* is *low* THEN *salary* is *low*

Fuzzy reasoning example (cont'd)

<u>Inferencing value of solution variable *salary*</u>

Given membership of *years_in_job* in set *high* = 0.5,
the contribution of the rule

IF *years_in_job* is *high* THEN *salary* is *high*

to making salary *high* will be to a degree of 0.5

- Truth values of all rules contributing to the membership of *salary* in *high*, are combined using the *min-max rule* to give the aggregate truth value for high salary

Fuzzy reasoning example (cont'd)

- Other rules give truth values for propositions *salary* is *medium* and *salary* is *low*
- The ultimate solution value of the variable *salary* is also determined through a combination process
- Combination of the fuzzy spaces for high, medium and low salary creates an *aggregated fuzzy region*
- A *defuzzification process* computes the numerical output value for salary from the aggregated fuzzy output region

The Min-max rule

- Fuzzy rules of inference are used to combine the fuzzy regions produced by the application of many rules run in parallel

- The most common method for this combination process is the *min-max rule*:

  The composite membership value of the LHS is the minimum of the memberships of all of the conditions on the LHS

- Example: Given the rule

  IF *a* is *X AND b* is *Y* THEN *c* is *Z*

- If the membership value of *a* in *X* is 0.5, and that of *b* in *Y* is 0.2, the degree of truth of the consequent (membership value of *c* in *Z*) will be min(0.5,0.2) = 0.2

The Min-max rule (cont'd)

- <u>If a number of rules lead to different membership values for an output variable, the maximum of these values is taken as the membership value.</u>

- Given a number of rules producing different truth values T1, T2, .., Tn for the membership of *c* in *Z*, the aggregated truth value is maximum(T1, T2, .., Tn)

- The following rules lead to differing membership values (shown in parentheses) for the output variable *risk* in the fuzzy set *high*,

  IF *age* is *middle* THEN *risk* is *medium* (0.3)

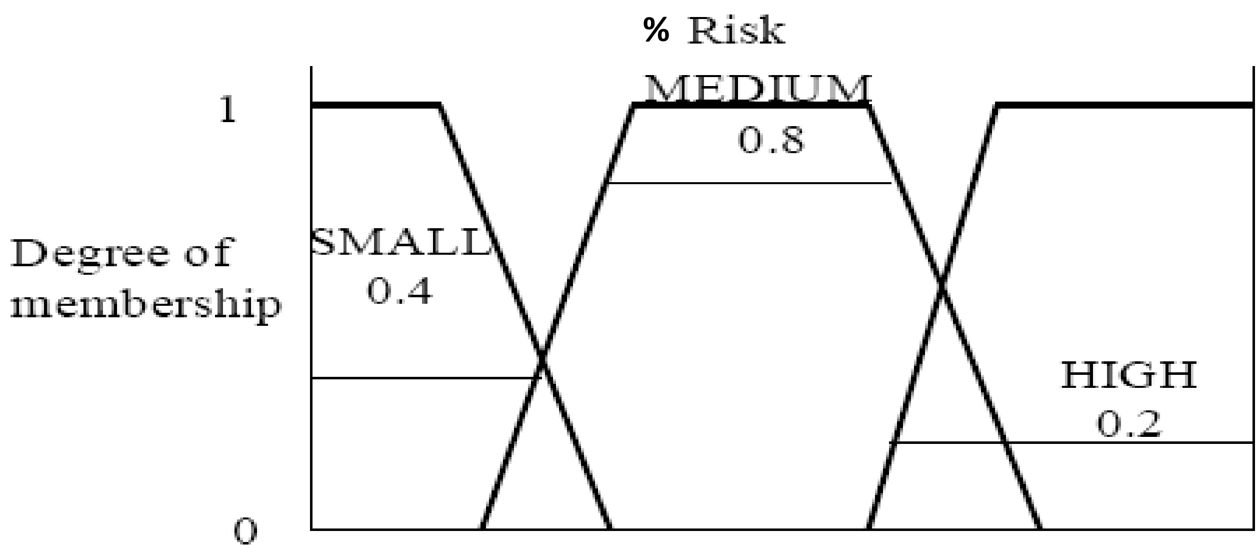
  IF *asset* is *medium* THEN *risk* is *medium* (0.2)

  IF *credit_history* is *reasonable* THEN *risk* is *medium* (0.8)

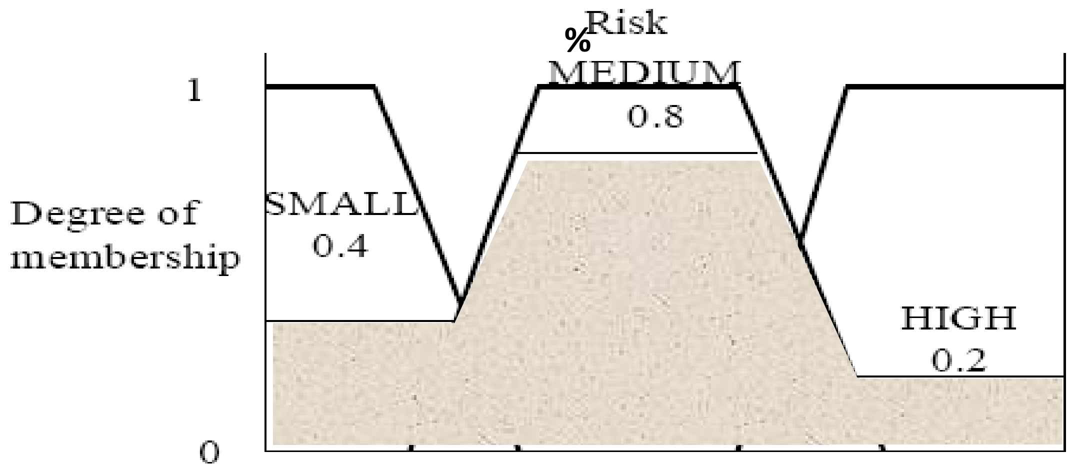
- Variable *risk* will have a membership value of max(0.3, 0.2, 0.8) = 0.8 in *medium*.

Defuzzification

- With the application of a number of rules for the person in the above example, the values for his/her membership in the *small* and *high* sets will also be similarly evaluated using the min-max rules
- Suppose these values are 0.4 for small, and 0.2 for high
- These membership values will truncate the fuzzy spaces for the sets *small*, *medium* and *high* as shown below



Defuzzification (cont'd)



Fuzzy spaces truncated by membership values for the sets small, medium and high

- These fuzzy regions are combined to give the aggregated fuzzy space for the output variable *risk*
- The numerical value for *risk* is computed from the aggregated fuzzy space by *defuzzification*

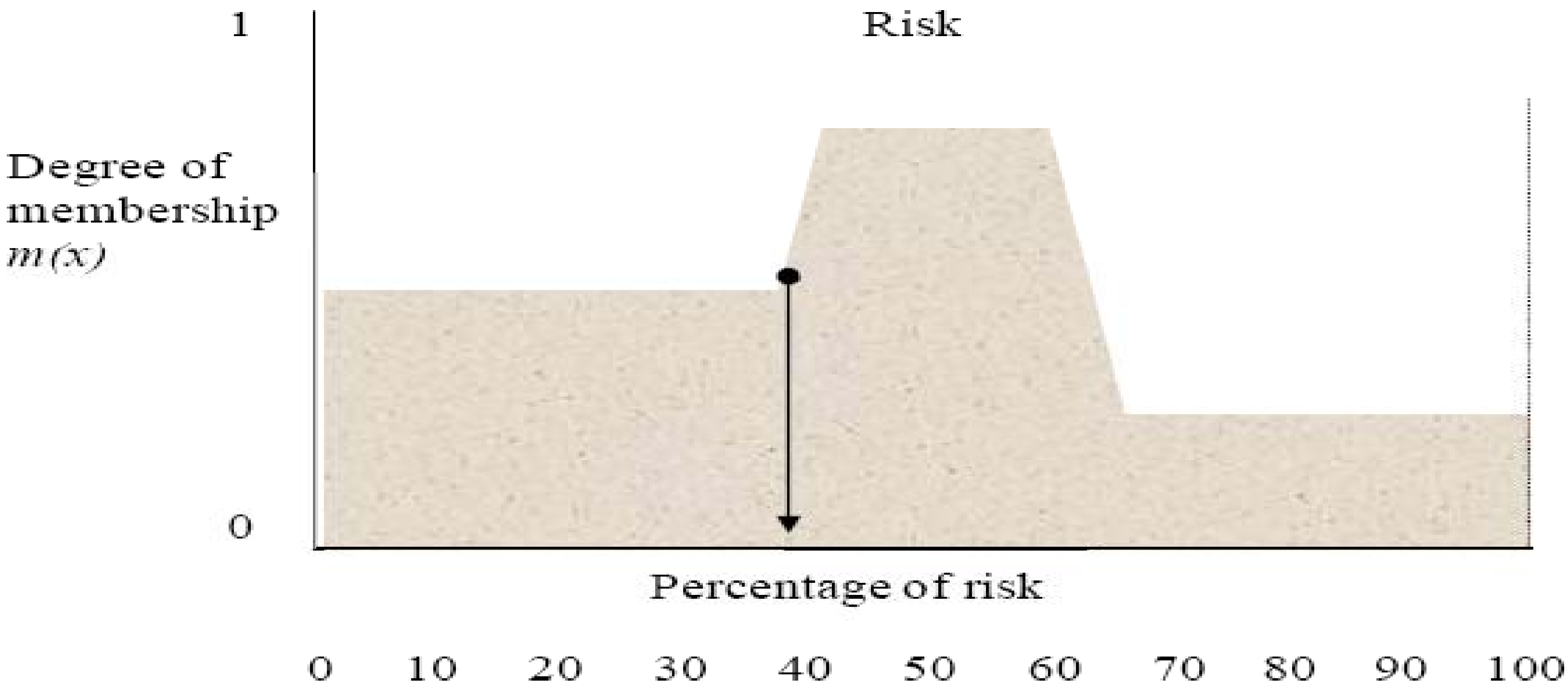
Defuzzification (cont'd)

- Defuzzification assigns an exact numerical value to the aggregated fuzzy region for the output variable
- The most common defuzzification method is the *centroid* or *centre of gravity* method
- It is a weighted average *R* of the output membership function:

$$R = \frac{\sum_{i=0}^{n} d_i . m(d_i)}{\sum_{i=0}^{n} m(d_i)}$$

Were d_i is the *i*th value along the horizontal axis, *n* is the maximum value of the range on the horizontal axis and $m(d_i)$ is the membership value for that point

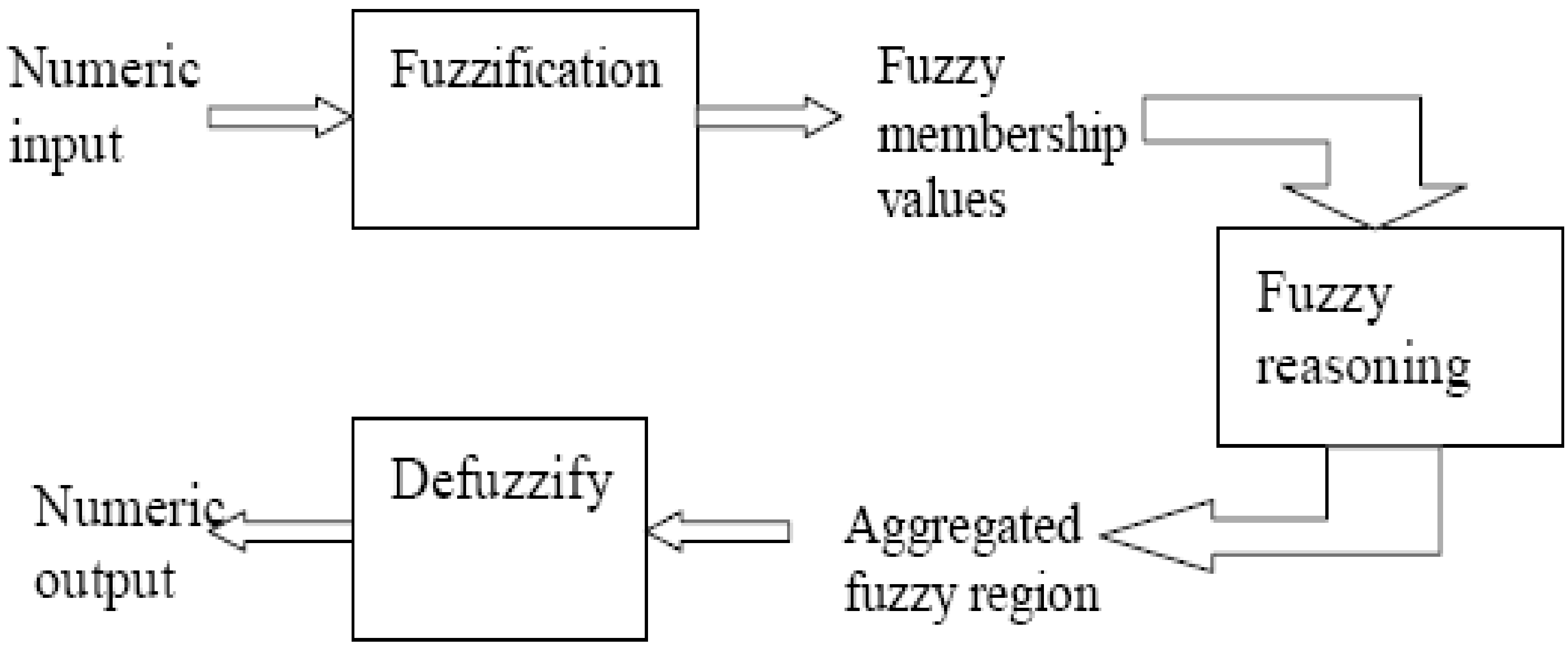
Defuzzification (cont'd)



The centroid method for calculating a fuzzy systems output value.

Fuzzy system operation - an overall view

The operation of a fuzzy system is shown in the schematic diagram below.



Design of a fuzzy controller

- Actions of a fuzzy controller are defined by a rule base
- Five steps in the construction of this rule base:

1. Identify and list the input variables and their ranges,
2. Identify and list the output variables and their ranges,
3. Define a fuzzy membership function for each of the input and output variables,
4. Construct the rule base that will govern the controller's operation,
5. Determine how the control actions will be combined to form the executed action.

Fuzzy controller design - a simplified example

- Controller to be used to smoothly slow and stop a train travelling at any speed and at any distance from station
- Step 1: Identify and list linguistic input variables and their ranges
  - Two input variables: train speed and distance to station
  - Five ranges each of speed (km/hr) and distance (m)

Fuzzy controller design - a simplified example

SPEED

Range of linguistic values:	Low	High
Fast	40	120
Medium Fast	10	50
Slow	2	15
Very Slow	0	4
Stopped	0	0

DISTANCE

Range of linguistic values:	Low	High
Far	2,000	∞
Medium Far	100	3,000
Near	5	200
Very Near	0	10
At	0	1.5

Fuzzy controller design - a simplified example (cont'd)

- **Step 2: Identify and list linguistic output variables and their numeric ranges**
  - Two input variables: train throttle and train brake
  - Five ranges each of train throttle (%) and brake (%):

THROTTLE

Range of linguistic values:	Low	High
Full	80%	100%
Medium	25%	85%
Slight	5%	30%
Very Slight	0	7%
No	0	0

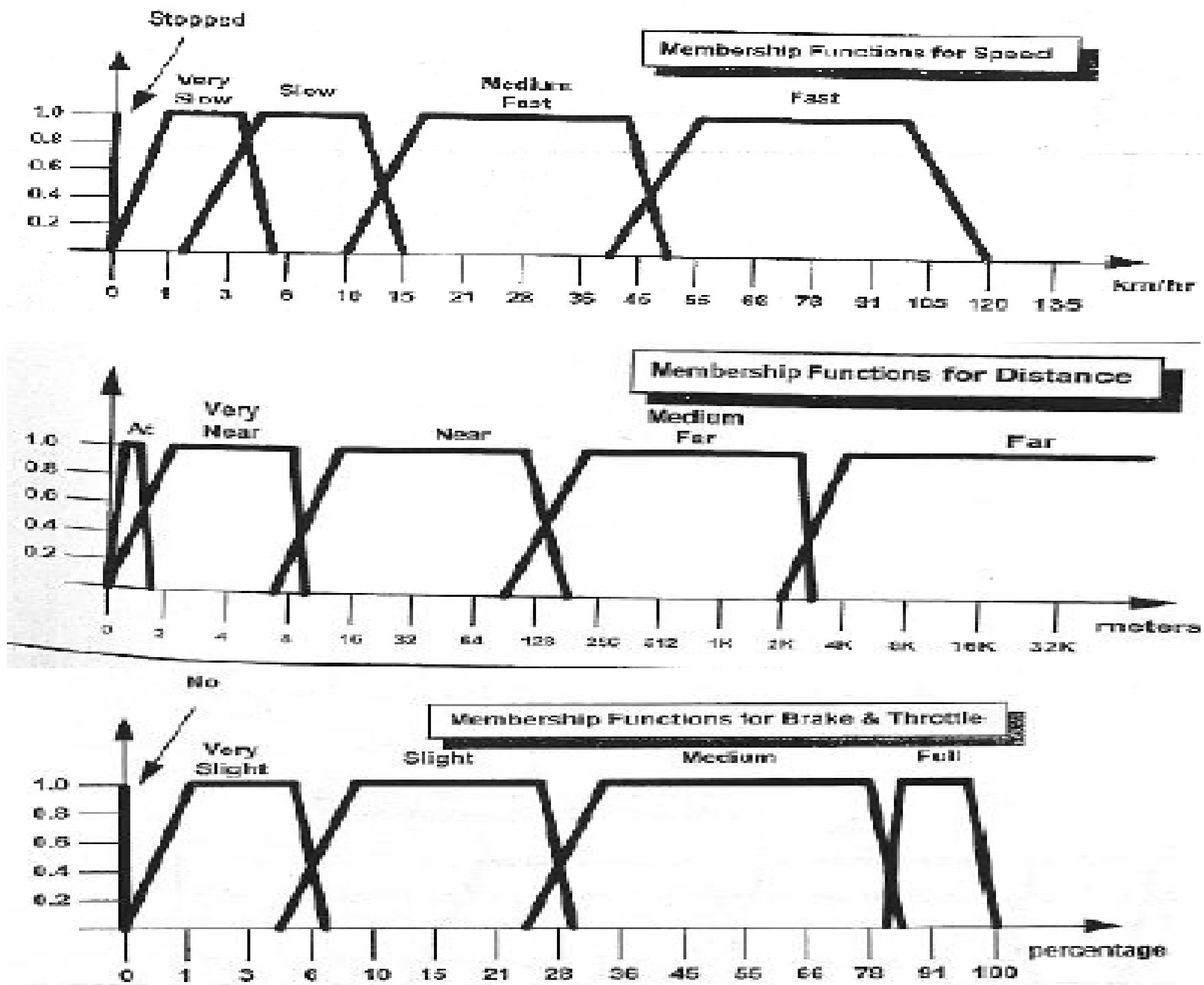
Fuzzy controller design - a simplified example (cont'd)

BRAKE

Range of linguistic values:	Low	High
Full	80%	100%
Medium	25%	85%
Slight	5%	30%
Very Slight	0	7%
No	0	0

- **Step 3: Define a set of fuzzy membership functions for each of the input and output variables**
  - Low and high values are used to define trapezoidal membership functions for each of the input ranges
  - Height of each function is 1.0 and function bounds do not exceed high and low ranges listed for each range

Fuzzy controller design - a simplified example (cont'd)



Fuzzy controller design - a simplified example (cont'd)

- **Step 4: Construct rule base that will govern controller's operation**
  - Rule base is represented as a matrix of combinations of each of the input range variables
  - Each matrix entry contains each of the two output range variables related to the input variables
  - Rule base matrix for example problem has only 12 rules that describe the interaction between input and output variables
  - Each entry in rule base is defined by AND-ing together the inputs to produce each individual output response.

Fuzzy controller design - a simplified example (cont'd)

- In the example diagram below, the shaded matrix entry means
  - IF *speed* is stopped AND IF *distance* is at THEN full brake
  - IF *speed* is stopped AND IF *distance* is at THEN no throttle

Distance / Speed	At	Very Near	Near	Medium Far	Far
Stopped	Full Brake No Throttle	No Brake V.S. Throttle			
Very Slow	Full Brake No Throttle	Med Brake No Throttle	No Brake Sl. Throttle		
Slow	Full Brake No Throttle	Mod Brake No Throttle	No Brake V.S. Throttle		
Medium Fast				No Brake Med Throttle	No Brake Full Throttle
Fast				No Brake Med Throttle	No Brake Full Throttle

Fuzzy controller design - a simplified example (cont'd)

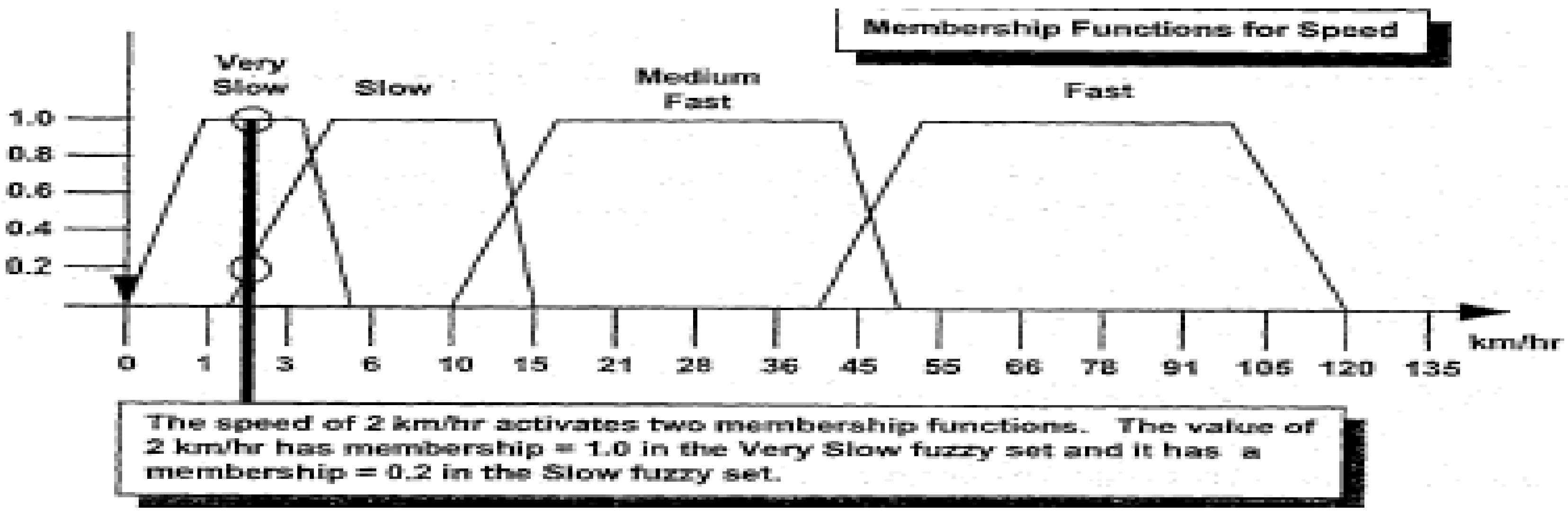
- **Step 5: Determine how control actions will be combined to form the executed action at the action interface**
  - Centroid defuzzification used for rule combination procedure
  - Consider the inputs:
    speed = 2 km/hr   and distance = 1 m
  - What is the correct % brake and % throttle?
  - First task: Determine which membership functions are activated and to what degree
  - Four membership functions are activated:
    the speed functions for *Very Slow* and *Slow*
    the distance functions for *At* and *Very Near* and

Fuzzy controller design - a simplified example (cont'd)

- Membership of the speed = 2 km/hr in fuzzy set for *Very Slow* is 1.0
- Membership of the speed = 2 km/hr in the fuzzy set for *Slow* is 0.2. Mathematically, they are denoted as

$M_{Very\ Slow}(2) = 1.0,$

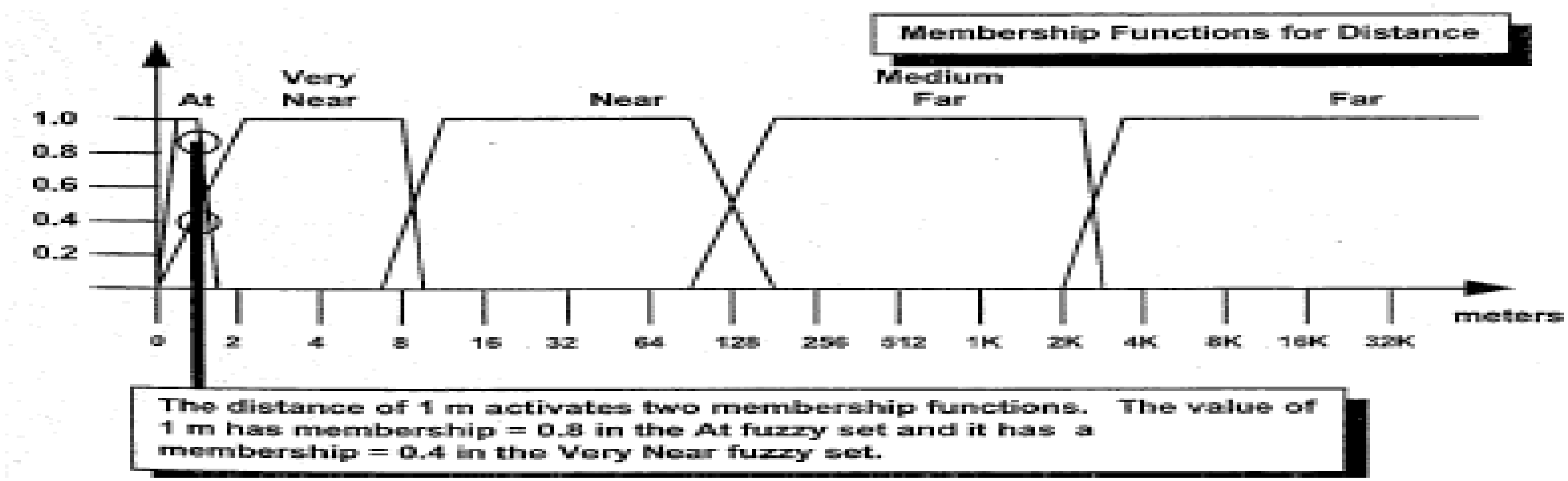
$M_{Slow}(2) = 0.2.$



Fuzzy controller design - a simplified example (cont'd)

- Similarly,
- membership values for the distance = 1 m in the fuzzy set for *At* and *Very Near* are:

$$M_{Very\ Near}(1) = 0.4,$$
$$M_{At}(1) = 0.8.$$



Fuzzy controller design - a simplified example (cont’d)

- This results in four rules firing in the rule base matrix

Speed \ Distance	At	Very Near	Near	Medium Far	Far
Stopped	Full Brake No Throttle	No Brake V.S. Throttle			
Very Slow	[illegible]ottle 1	No Th[illegible] 3	No Brake Sl. Throttle		
Slow	[illegible]rake 2	Med B[illegible] 4	No Brake V.S. Throttle		
Medium Fast				No Brake Med Throttle	No Brake Full Throttle
Fast				No Brake Med Throttle	No Brake Full Throttle

Fuzzy controller design - a simplified example (cont'd)

- Next, membership values are combined using the AND (min) operator for each rule combination:

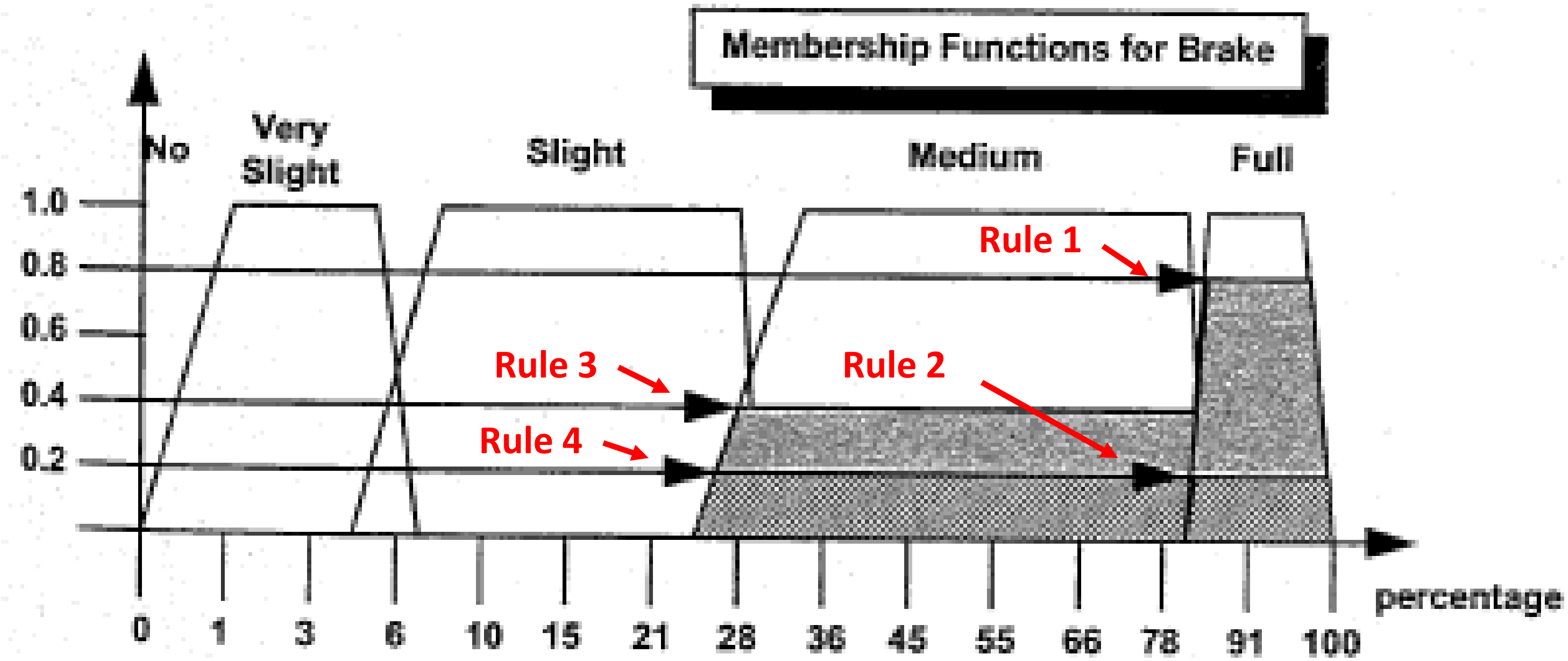
- Rule 1: $M_{VerySlow}$ AND M_{At} = min(1.0, 0.8) = 0.8,
- Rule 2: M_{Slow} AND M_{At} = min(0.2, 0.8) = 0.2,
- Rule 3: $M_{VerySlow}$ AND $M_{VeryNear}$ = min(1.0, 0.4) = 0.4,
- Rule 4: M_{Slow} AND $M_{VeryNear}$ = min(0.2, 0.4) = 0.2.

- The values 0.8, 0.2, 0.4 and 0.2 are the firing strengths of rules 1 to 4, respectively, for the input (2,1).

- Next, output value for each rule is determined by truncating the corresponding output membership function using its firing strength

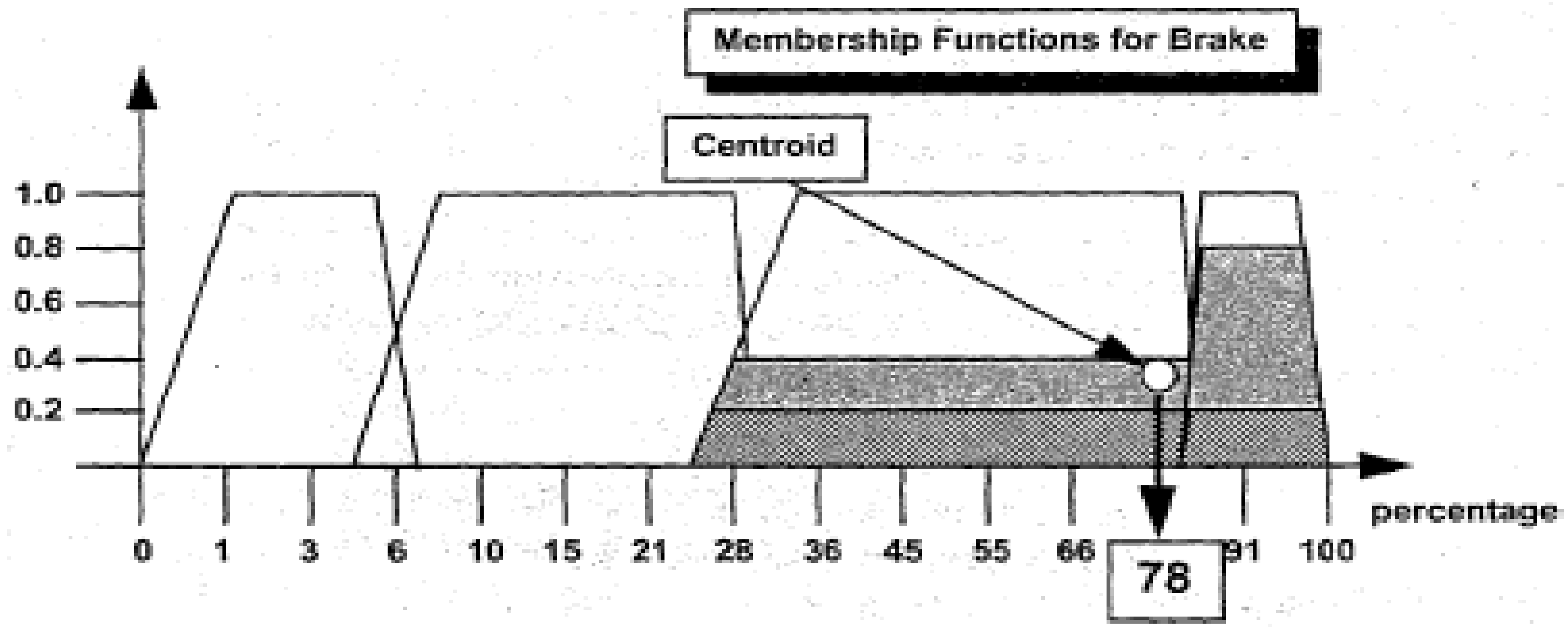
Fuzzy controller design - a simplified example (cont'd)

- The resulting aggregated fuzzy output region for the rules for variable *brake* :



Fuzzy controller design - a simplified example (cont'd)

- Finally, defuzzification using centroid method yields output value of 78 percent application of the brake



Fuzzy Controller Operation

- During operation, input values are continually sampled and presented to the fuzzy controller

- The fuzzy controller then repeats the process described above in Step 5:

- Determine the fuzzy membership values activated by the inputs
- Determine which rules are activated (fired) in the rule base matrix
- Combine the membership values for the activated rules using the AND operator

- Determine the aggregated fuzzy region for each output variable
- Use defuzzification to compute the values for each output variable

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